Mohamad Bornat | Root Locus | Control Systems I

Ex ⑥ Plot the root locus of the following open-loop transfer function:-

$$G(s) = \frac{K}{s(s+3)(s^2+2s+2)}$$

→ Zeros = ϕ

Poles = 0, −3 , $-1 \mp j$ ⟵ from the denominator

⇒ # of Branches = 4

→ to find Assymptotes :-

$$\sigma_a = \frac{\Sigma \text{Poles} - \Sigma \text{zeros}}{n-m} = \frac{[0 + -3 + -1 + -1] - 0}{4-0}$$

$$= -1.25$$

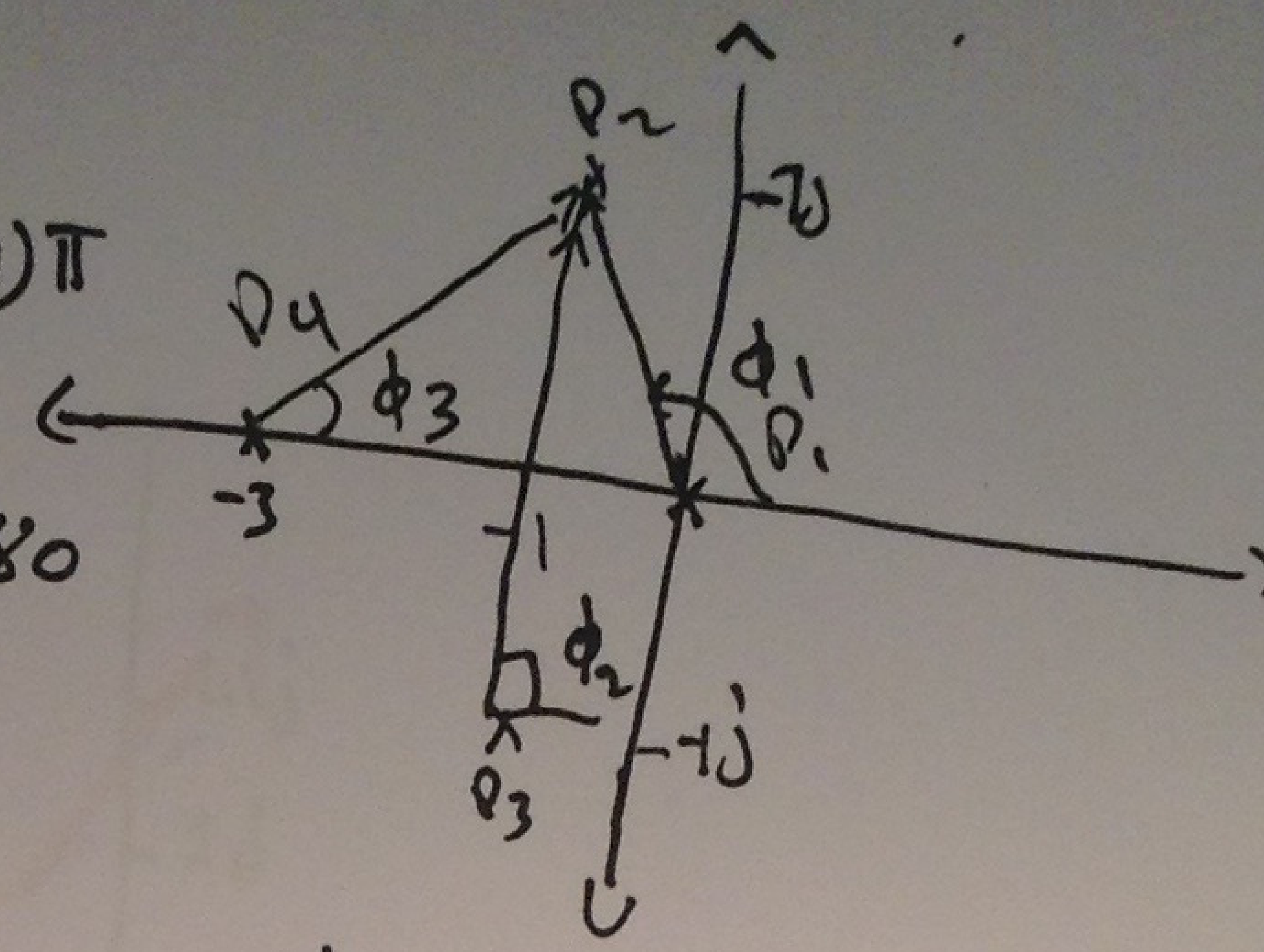
$$\theta_a = \frac{(2v+1)\pi}{n-m} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

→ to find Angles of Departure

$\measuredangle P_2 = \sum \measuredangle(Z_K + P_2) - \sum \measuredangle(P_2 + P_i) + (2v+1)\pi$

$= 0 - [180° - \tan^{-1} 3] - 90 - \tan^{-1} 0.5 + 180$

$= -72°$



$\measuredangle P_3 = 72$ due to symmetry.

the loci $\in]0, -3[$

→ To find Point of Branching

$\sum \frac{1}{s + P_d} = \sum \frac{1}{s + Z}$

$\cancel{=\frac{1}{s}}$ By solving $s = -2.4$

→ to find Crossing Points with Imaginary axis:

$1 + K\,G(s) = 0$

$\Rightarrow s^4 + 5s^3 + 8s^2 + 6s + K = 0$

* we use Routh-Hurwitz criterion
* Recall: ~~zero~~ row of Zeros Produce Points on the Im axis

s^4	1	8	K
s^3	5	6	0
s^2	6.8	K	0
s^1	6 - 0.73K	0	
s^0	K		

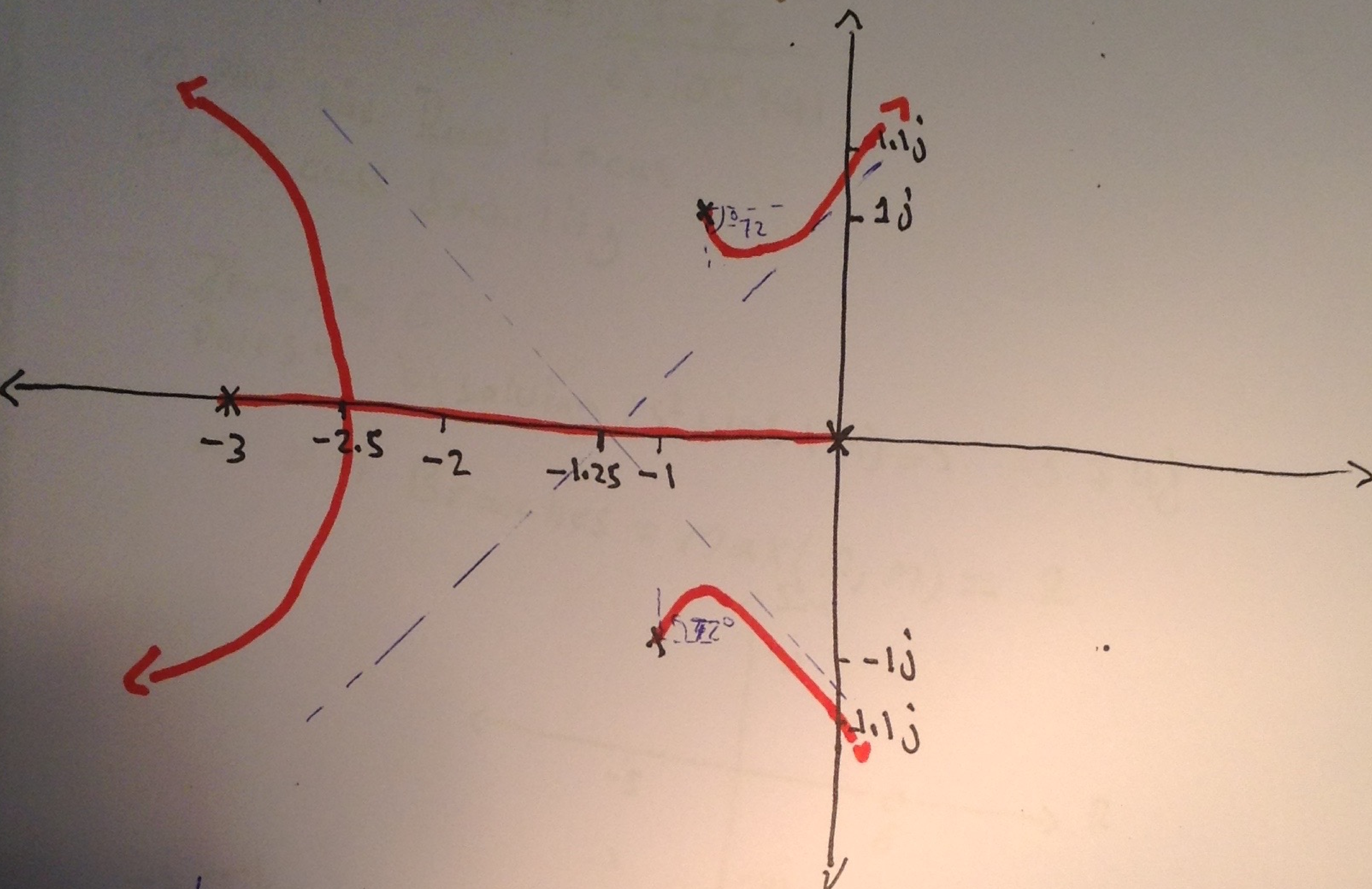
$\Rightarrow K_{critical} = 8.2$

Auxiliary equation:

$6.8 s^2 + 8.2 = 0$

$\Rightarrow s = \mp 1.1j$

The root Locus plot :



The system is stable for $0 < K < 8.2$

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Example (2) for the following open-loop unity negative feedback transfer function:-

$$G(s) = \frac{s-6}{s^2+10s+41}$$

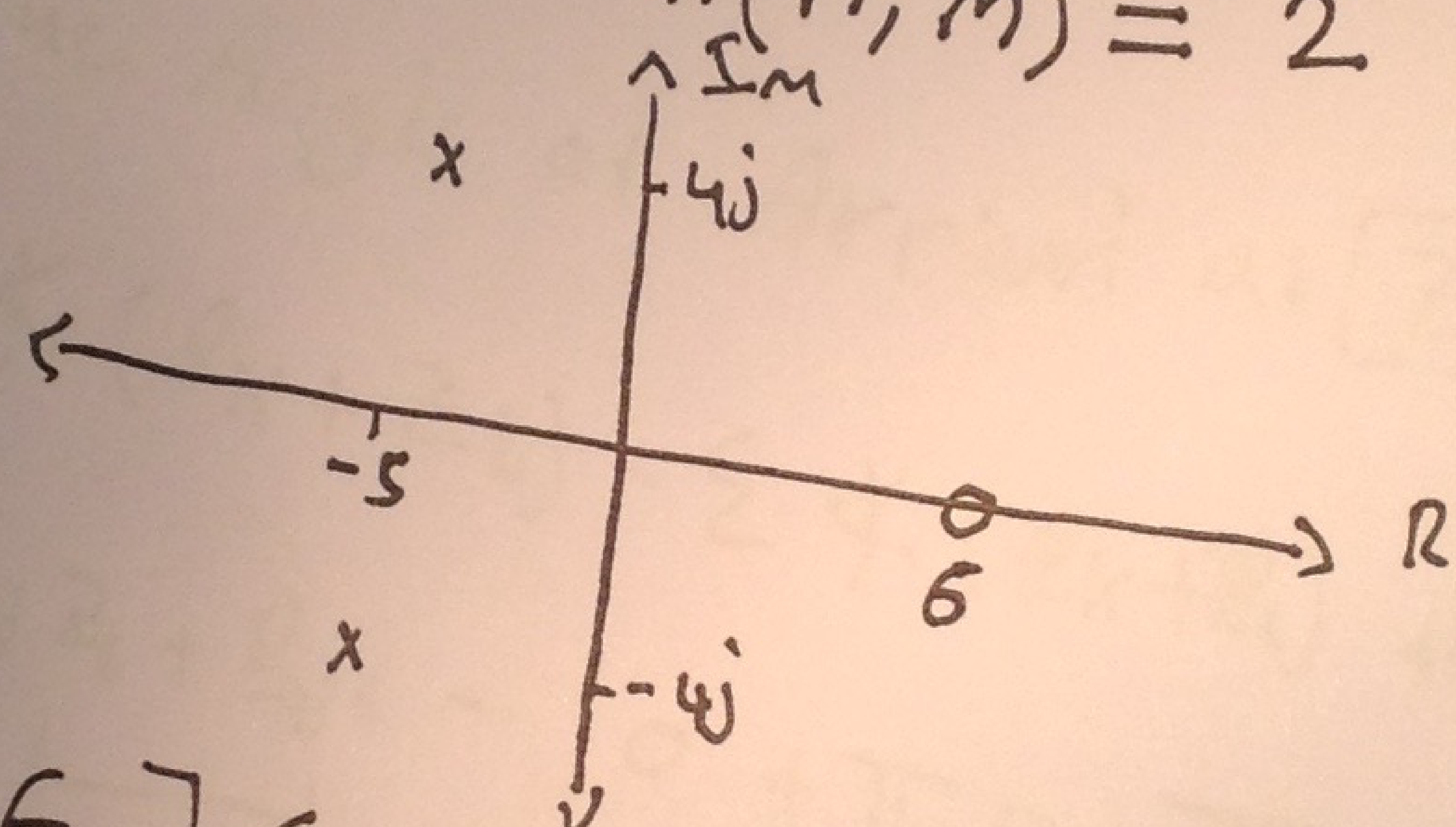
(1) Plot the Root Locus

(2) Discuss Stability

→ Zeros: 6

Poles: by solving $(s^2+10s+41) \Rightarrow -5 \mp 4j$

⇒ # of Branches = Max(n, m) = 2



→ The loci $\in \,]6, -\infty[$

→ to find Assymptotes

$$\sigma_a = \frac{\Sigma P - \Sigma Z}{n-m} = \frac{(-5+-5)-6}{2-1} = -16$$

$$\phi_a = \frac{(2v+1)\pi}{n-m} = \pi$$

Not useful informations for this example.

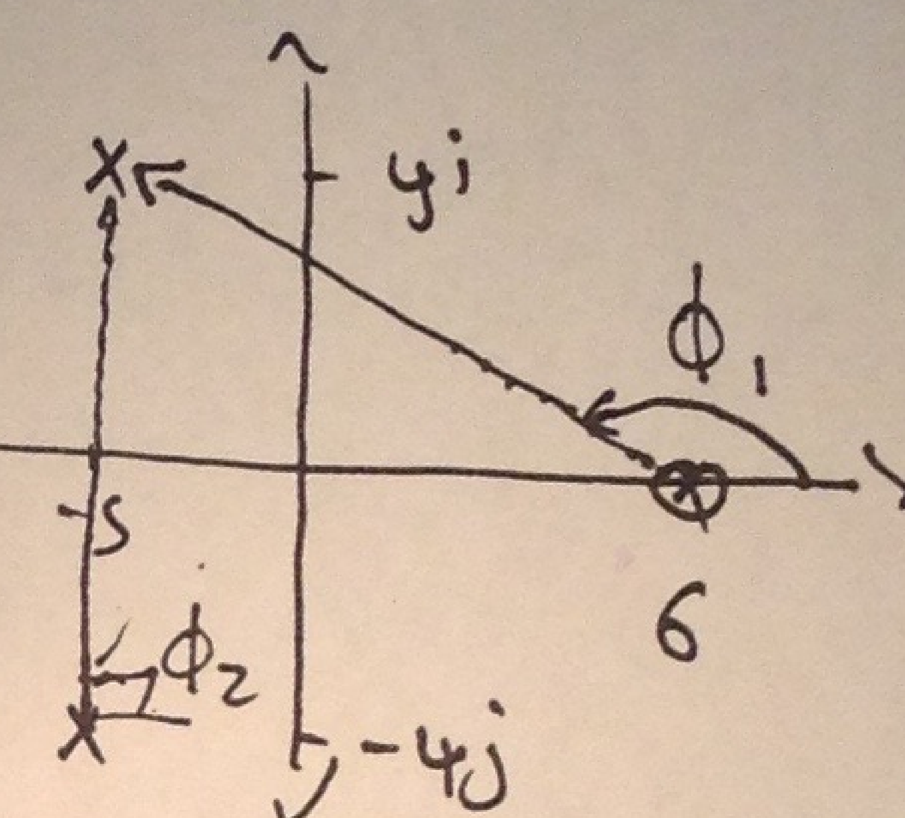
→ to find the Angles of Departure

$$\measuredangle P_i\Big|_{i=-5+4j} = \sum \measuredangle (z_k + p_i) - \sum (p_k + p_i) + (2v+1)\pi$$

$$= \left[180 - \tan^{-1}\left(\tfrac{4}{11}\right)\right] - 90 + 180$$

$$= 250^\circ$$

$$\measuredangle P_i\Big|_{i=-5-4j} = -250^\circ$$ due to symmetry.



→ to find the Angle of Arrival at $\boxed{z=6}$

$$\measuredangle \phi_{z_i}\Big|_{i=6} = \sum \measuredangle (p_k + z_i) - \sum \measuredangle (z_k + z_i) + (2v+1)\pi$$

$$= \underbrace{\measuredangle P_{4j}\, z_6 + P_{-4j\,6}}_{\text{cancel each other}} - 0 + \pi = \pi$$

→ to find the Point of Branching

$$\sum \frac{1}{s+p_i} = \sum \frac{1}{s+z_i}$$

$$\frac{1}{s+5-4j} + \frac{1}{s+5+4j} = \frac{1}{s+6}$$

By Solving: $s = 17.5$ ✗ because it $\notin L$

or

-5.75 ✓

→ to find the Crossing Point/s with the Imaginary axis:-
we should use Routh-Hurwitz criterion

$1 + K\,G(s) = 0$

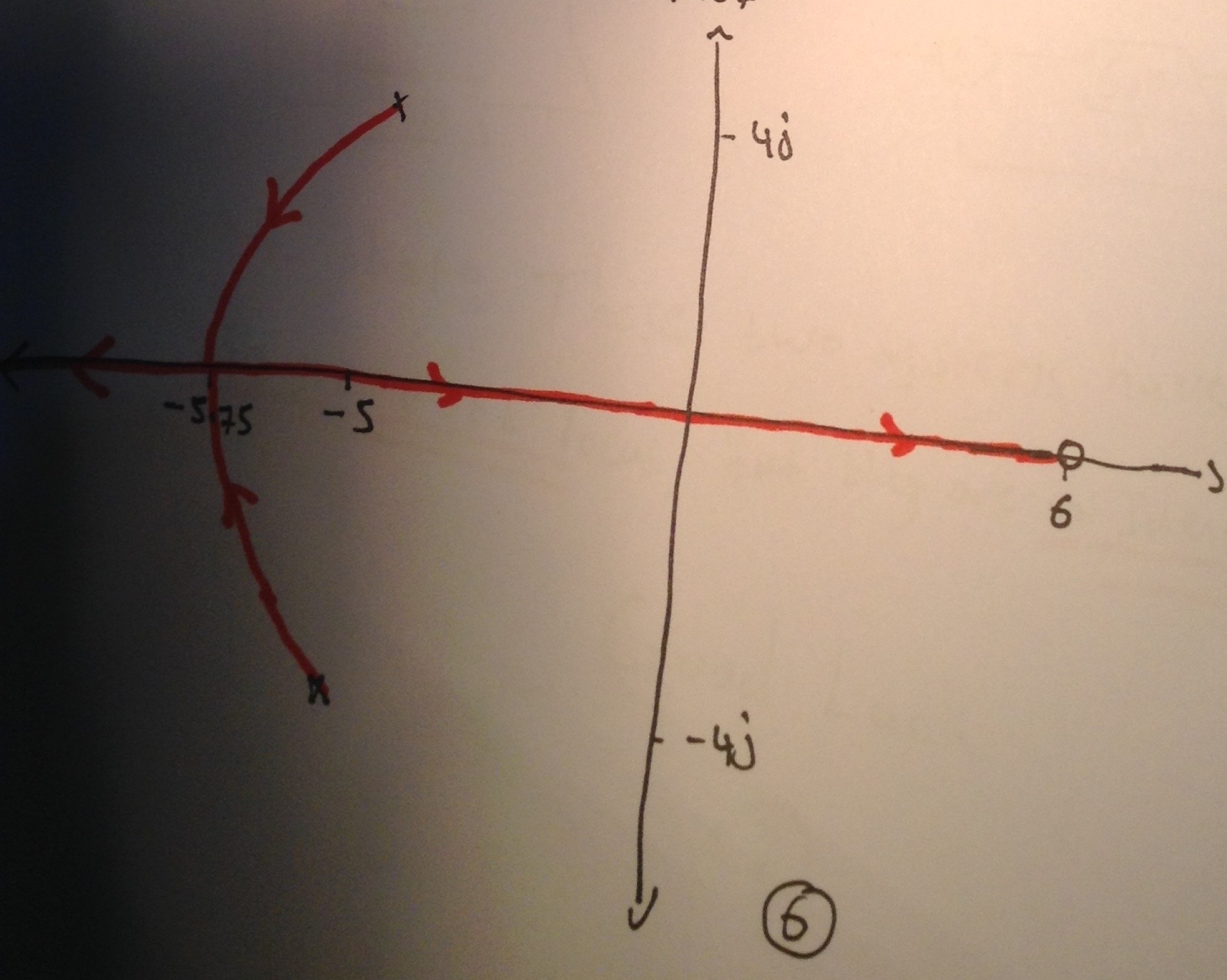
$\Rightarrow s^2 + (10+K)s + (41-6K) = 0$

s^2	1	$-6K+41$
s^1	$10+K$	0
s^0	$41-6K$	0

* Recall: row of zeros produce "points" on the Im axis.

$\Rightarrow$ for $K>0$ $\quad K < 6.8$

→ The Root locus Plot



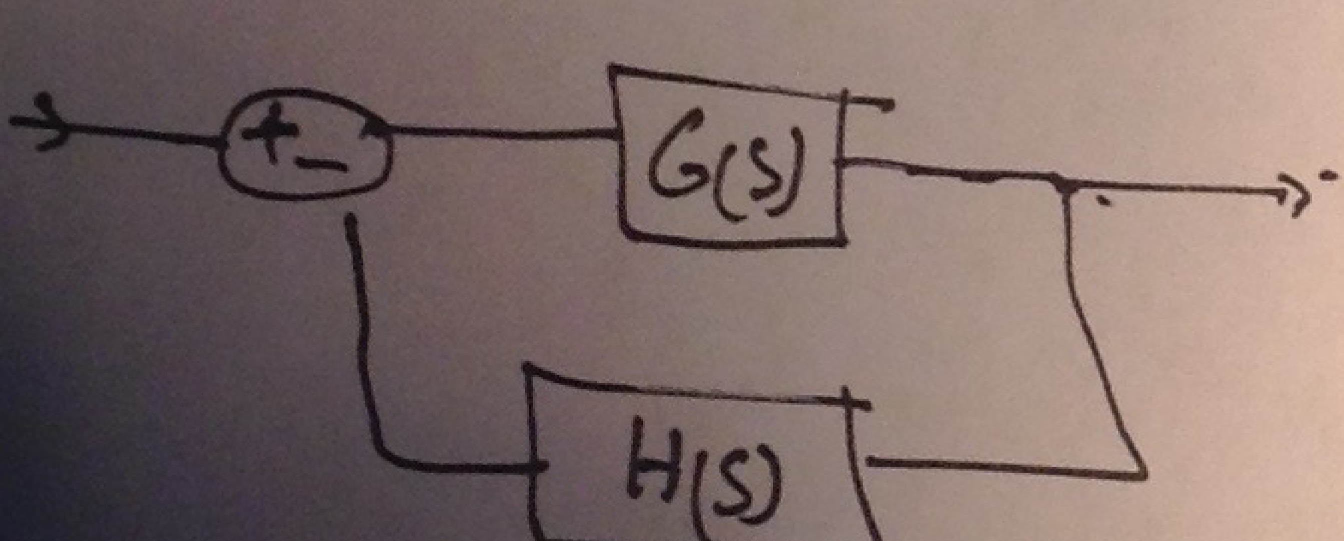
* What if?

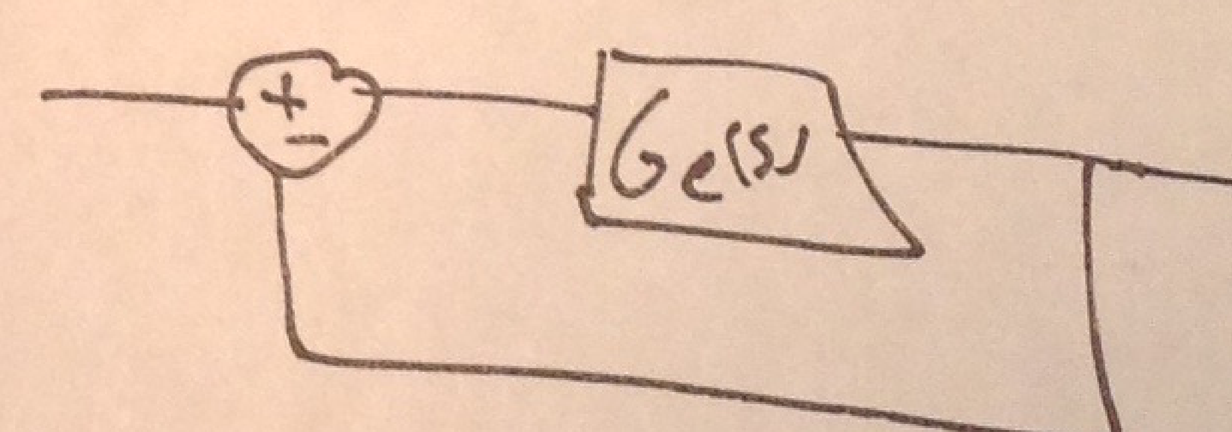
What if you are given a non-unity feedback system? with $H(s)$ in the feedback path.

→ Then for root locus sketching we should consider $G_e(s) = G(s) \cdot H(s)$ as the function to get zeros & poles.

Direct Path TF

feedback Path TF





* Note that: These two systems have the same root locus, but they are not Identical systems.

Good Luck